The Derivative Markets: an Overview

What is a derivative?

A derivative is a financial asset that represents a claim to another financial asset. Derivatives are found in a wide range of markets but the more common are the interest rate markets, the foreign exchange markets, the equity markets, the commodity markets and the credit markets.

Simply put, a derivative can be classified in one of two ways. It will either be a *forward* product (an agreement between two parties requiring the sale of an asset or product in the future at a price agreed upon today), an *option* product (an agreement between two parties giving the option buyer the right, but not the obligation, to buy or sell an asset in the future at a price agreed upon today), or a combination of the two. The subject of derivatives is viewed by many with suspicion and is often thought of as being complex and hard to understand, but at the end of the day, any derivative product – regardless of fancy title - can be decompounded into one of these two product families.

For example, the *forward* family includes products such as *forward rate agreements*, *futures*, and *swaps*. These are all examples of contracts that can be bought and sold in order to lock in pre-agreed forward prices or rates to be paid by the buyer to the seller on the settlement date. The buyer benefits if prices increase because the buyer has locked in at a lower price, and the seller benefits if prices fall because a higher selling price has been locked in. Therefore, only one party to a forward contract can win at the expense of another, so the forward contract is a zero-sum game.

On the other hand, the *option* family includes products such as *calls* and *puts*, *caps* and *floors*, c*ollars* and *swap options*. Option products differ in two respects from forward products: firstly, whereas a forward represents an obligation to transact, an option is a right, but not an obligation, to deal at a pre-agreed price or rate; secondly whereas a forward involves no up-front cost to either party, an option contract requires the option buyer to pay a sum of money called the premium to the option seller.

At this stage, it should be noted that derivatives trade in both over-the-counter (“OTC”) markets and the exchange-traded markets. The OTC market describes the informal trading that goes on between counterparties who deal over the phone directly with each other. They result in bilateral deals with a range of different terms and conditions. This compares with the exchange-traded contracts that are agreed between two parties transacting in an exchange. Exchange transactions are governed by the rules of the exchange that typically standardize the terms of the contract (in terms of amount, maturity etc.), and require each the exchange to take an intermediary position against each contract. In addition, to ensure the creditworthiness of each counterparty, the exchange will insist on “margining” whereby gains and losses are monetized at least on a daily basis.

While a detailed discussion of these products is beyond the scope of this text, we will explain briefly the conventions and mechanics associated with the basic products, offer an insight into some applications and briefly discuss pricing in the sections that follow. The focus will be on interest rate derivatives, but it should be noted that the same concepts apply to all markets: whether we are talking about interest rates, or FX or equity, the operation and pricing or each type of swap or option, is broadly similar.

Let us look at the basic products in turn, starting with those of the forward family, and concluding with a description of option product. *Note: the rates in all the examples that follow are the actual rates reported as at March 2013.*

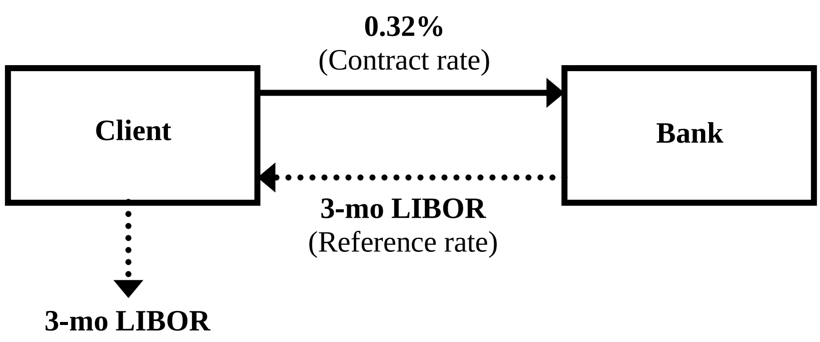
Forward Rate Agreements

A Forward Rate Agreement ("FRA") is a derivative product that, as its name suggests, locks in a future interest rate for a period of time. FRAs are OTC instruments offered by banks to their customers to allow for hedging against or to speculate on a movement in future interest rates.

Mechanics of an FRA: an example:

- A client who is borrowing $10 million against 3-month LIBOR in 3 months time wants to protect against rising rates and requests a price for a 3 x 6 FRA. The terminology used describes an FRA that fixes a three month rate starting in three month's time: the first number ("3") defines the start-date in months, and the second number ("6") defines the end-date in months.
- The bank quotes 0.32% - and the client, as notional borrower, deals.
- Settlement under the contract will take place in 3 month's time ie. at the beginning of the contract when LIBOR sets. There will be no exchange of principal, but rather a cash settlement based on the difference between the contract rate (the FRA rate agreed in advance) and the reference rate (being whatever the new 3-mo LIBOR rate is in 3 month's time) on the notional amount over the time period. The cash flows can be represented with arrows in **Figure 1** below, whereby an arrow going away from the box represents an outflow while arrows coming in represent cash inflows

Figure 1: Cash flows under an FRA settlement



- Gains or losses under the FRA contract will offset the respective "losses" or "gain" under the loan, thereby always ensuring that the investing client pays 0.32% for 3 month money in three month's time - no more or no less.

FRA Pricing: an intuitive approach

The pricing of an FRA, like any standard forward, is a mechanical function driven by interest rate differentials.

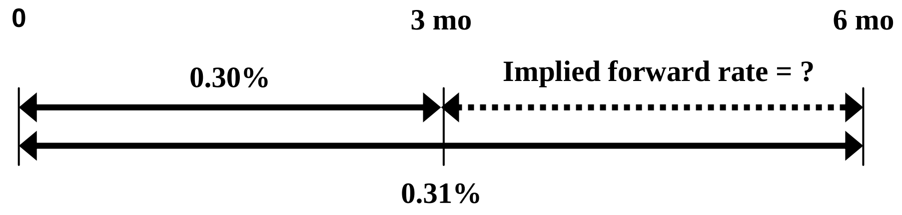
Assume now an investor, now with a 6-month time horizon, has the following choices:

1. invest for six months at 0.31%, or

2. invest for three months at 0.30%, and then reinvest the proceeds at some forward rate that can be locked in using a 3 x 6 FRA.

What is the implied forward rate?

Figure 2: The 3 x 6 FRA



The implied forward rate – the FRA rate - is basically a no-arbitrage equilibrating rate that can be *approximated* by determining the average rate which would make the investor indifferent between investing for 6 months at 0.31% or investing for 3 months at 0.30% and then reinvesting the proceeds at that to-be-determined forward rate. Ignoring the impact of compounding, our break-even, no-arbitrage forward rate is calculated to be 0.32% because only 0.30% plus an implied 0.32% rate will give us an arithmetic average of 0.31%. (**Note:** taking into account the compounding effect to determine the precise value gives us a number that differs at only the fourth decimal place, so we can ignore it in this intuitive overview of forward pricing. Such are the mechanics of the FRA product, one of the simplest of derivative products, being a single-period forward contract that will lock the counterparties into a fixed rate for a designated period of time at zero up-front cost.

Futures

A futures contract is an exchange traded forward contract. They are routinely bought and sold in a remarkable variety of items, but the most liquid markets exist in commodities and financial futures. The contracts themselves are traded in a variety of exchanges around the world which may be either based in a physical location, or, more frequently these days, traded via a set of interconnected screens. Trading has to be done though a firm that is a member of the exchange in question: the instructions are routed through the member who will act in an agency capacity and transact with other exchange members who may in turn be dealing on behalf of other counterparties. Deals, when done, are then confirmed to the initiating party.

As mentioned, exchange-traded futures contracts differ from OTC products insofar as the following is concerned:

1. Whereas OTC contracts are tailored to meet the precise needs of the counterparties, the futures contracts are standardized by way of deliverable grade, maturity and amount in order to concentrate the counterparty interest and promote liquidity

2. Whereas OTC contracts involve bilateral contracts between two counterparties, in a futures transaction, once the deal is agreed between two counterparties, the exchange intermediates and becomes the buyer for every seller, and the seller to every buyer.

3. Whereas OTC contracts typically expose the counterparties to the credit risk of the other, in an exchange-traded transaction, the risk is mitigated somewhat because of the *margining* process. By this, the exchange requires both counterparties to deposit an *initial margin* (representing a percentage of the underlying notional) with the exchange from the time they enter into a contract, and then the daily mark-to-market is crystalised through the payment (in the event of a loss) or receipt (in the event of a gain) of a *variation margin*.

4. In excess of 90% of contracts are terminated before expiry (effectively, by doing another deal to offset the initial trade) whereas most OTC deals run their full course.

As mentioned, futures are offered on a huge range of financial products including bonds, FX rates, and a wide number of equity indices and individual stocks. However to demonstrate how a typical futures contract operates, let us look, by way of example, at the *Eurodollar future,* the exchange-traded equivalent of the OTC 3-month FRA in order to get a sense of the how a typical futures contract operates. Like the FRA, it is a single-period forward contract that could serve to hedge a borrowing client from higher interest rates going forward in time.

The Eurodollar Futures Contract

The Eurodollar future, traded on the CME, is one of the most liquid futures contracts in the world, the underlying "asset" being the 3-month LIBOR rate. You can see from the terms and conditions for the contract shown in **Figure 4** that "standardization" of the contract extends to its notional value ($1,000,000) and maturity (the third Wednesday of every third month – although they do trade every month in the short-term). You will also observe that the value of a tick, being 1 basis point or the minimum price move, is $25 calculated as $1,000,000 x 1 basis point over 3 months ($1,000,000 x .0001 x 3/12).

Figure 4: Terms and Conditions of the Eurodollar Futures Contract

Unit if Trading:	$1,000,000
Delivery months:	March, June, September, December
Delivery date:	First business day after last trading day
Last trading day:	11:00am/ Third Wednesday of delivery month
Quotation:	100.00 minus interest rate
Minimum price movement:	0.01% (one tick)
Tick value:	$25.00

The prices in **Figure 5** below are those quoted for the Eurodollar future series back in March 2013 going out three years.

- The *Settle* is the settlement or closing price for the day.
- The *Yield* is the 3-mo LIBOR forward rate calculated as 100.00 minus the settlement price. For example, we can interpret 0.32% as being our 3-mo LIBOR rate out of June 2013 (ie. 3 month rate in 3 month's time), 0.35% as being the 3-mo LIBOR rate out of September 2013, 0.39% rate out of December 2013 and so on. As mentioned, one can draw the analogy with FRAs: these are the exchange-traded equivalents to respectively a 3 x 6 FRA, a 6 x 9 FRA and a 9 x 12 FRA, and all represent 3-mo forward rates for LIBOR. And they are priced in exactly the same way as FRAs using interest rate differentials, which of course is why the actual 3 x 6 FRA rate of 0.32% is exactly matched by the June 2013 future. Beware, however: the futures contracts are traded on a price (being 100 minus the rate) whereas FRAs trade on a rate

Figure 5: Eurodollar Futures Prices, March 2013

Month	Settle	Yield
Jun 13	99.68	0.32%
Sep 13	99.65	0.35%
Dec 13	99.61	0.39%
Mar 14	99.59	0.41%
Jun 14	99.56	0.44%
Sep 14	99.52	0.48%
Dec 14	99.47	0.53%
Mar 15	99.41	0.59%
Jun 15	99.34	0.66%
Sep 15	99.27	0.73%
Dec 15	99.20	0.80%

If we think about the position of our floating rate borrower mentioned earlier, the exposure is to higher rates in an underlying notional of $10 million. The borrower would therefore need to *sell* 10 June contracts in March 2013. No matter where LIBOR goes over the following three months, any "gains" or "losses" on the underlying loan resulting from respectively lower or higher rates, there will be an offsetting loss or gain on the futures, thereby ensuring that our borrowing client was effectively locked in at 0.32% - no more or no less.

Obviously if the borrowing exposure was longer term, then the client could hedge the exposure by selling additional futures contracts. If, for example, the loan was a 3-year loan which was repricing quarterly against 3-mo LIBOR, then the borrower would need to sell a series or a strip of futures contracts going out to December 2015; the initial LIBOR setting is known as it has set today, but we need to lock in next *eleven* LIBOR rates.

However, an easier solution for such a borrower who is exposed to higher interest rates for multiple periods over a longer time horizon is an *interest rate swap*.

Interest Rate Swaps

An *Interest Rate Swap* is an agreement between two counterparties to exchange a stream of cash flows at specified intervals. It can be viewed as a series of forward contracts: recall that an FRA or Eurodollar future enabled a borrowing client to fix a borrowing rate on a specific future date. With an interest rate swap, the only difference is that there are multiple exchanges instead of just one. In fact, if we know what the single-period forward contract rates are, we are in position to price an interest rate swap.

At the end of the last section on futures, it was suggested that if the borrowing exposure was three years, then the client could hedge the exposure by selling a strip of futures contracts going out to December 2015. Including today's rate, under such a transaction, the borrower could lock in the relevant rates for the appropriate periods (as per the futures table). In calculating the all-in-cost, one could get a good approximation by finding adding up the twelve different rates, and then dividing by twelve to achieve an arithmetic average of 0.50%. To be precise in one's calculations, one needs a time-weighted average rate (alternatively known as the *internal rate of return*). Although the calculation of this number is beyond the scope of this text, the number happens to be the same as the arithmetic average of 0.50%. This "average-of-the-forwards" rate is known alternatively as the (*interest rate*) *swap rate*.

An interest rate swap is an agreement between two parties to exchange a stream of cash flows on a predetermined set of dates in the future, based on a notional principal amount. One party to the transaction is the fixed rate payer while the other party is the fixed rate receiver. There is no exchange of principal, only exchange of interest.

To best explain the mechanics of an interest rate swap, let us look at typical swap pricing grid with rates dating from March 2013 shown in **Figure 6** below.

Figure 6: US dollar IRS rates, March 2013

Tenor	Column 1	Column 2	Column 3	Column 4
	UST rate	**Swap spread**	**Swap rate (sa)**	**Swap rate (qtly)**
2 Yr.	0.28%	.13	0.41%	0.40%
3 Yr.	0.39%	.12	0.51%	0.50%
5 Yr.	0.78%	.12	0.90%	0.88%
10 Yr.	1.80%	.05	1.85%	1.82%

Let us interpret these numbers.

Column 1: UST rate — This represents the yield for what is referred to as the "*on-the-run*" US Treasury bond ie. the most recently issued, liquid US T-bond

Column 2: Swap spread — This is the spread in basis points which is to be added to the US Treasury yield in order to determine the outright swap rate.

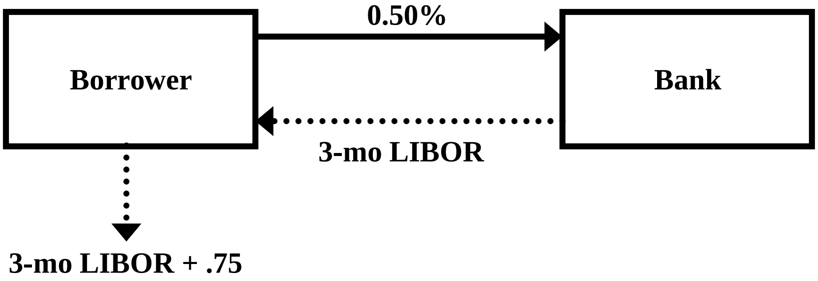
Column 3: Swap rate (sa) — This is the outright swap rate on the swap. The US Treasury instrument pays a semi-annual coupon so, by convention, the interest rate swap is quoted on a semi-annually yield basis.

Column 4: Swap rate (qtly)	This represents the outright rate on the swap quoted on a quarterly (money market) basis. The rate in *Column 3* is adjusted for the change in the day-count and the compounded return.

In the case of swaps, we need to reiterate that the outright rates (US Treasury yield plus the spread) represent the fixed rates which are paid or received versus the floating rate, the latter which is always, by convention quoted as LIBOR flat. With reference to the pricing grid above, we can determine the all-in-cost or net return as follows:

Example 1: A company, which has a 3-year floating rate loan repricing quarterly at a rate of LIBOR + 75 basis points, is concerned about rising rates, and wishes to pay a fixed rate. The all-in-cost to the borrower is 0.50% plus 75 basis points, or 1.25%, as indicated in **Figure 7(a)** below:

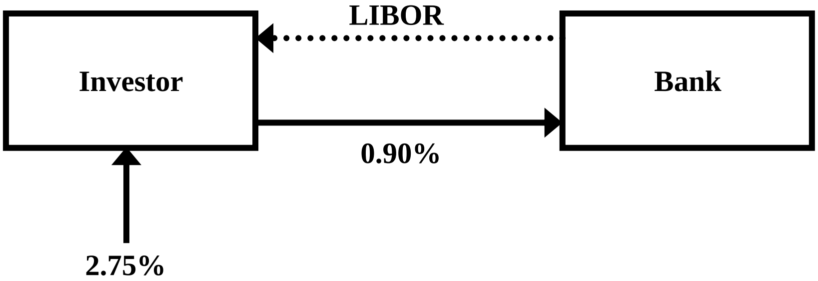
Figure 7(a): 3-year IRS



All-in-cost to borrower = 0.50% (fixed rate) + 0.75% (spread) = **1.25%**

Example 2: An investor, who owns a 5-year fixed rate bond that is paying a semi-annual coupon of 2.75% (an annualized rate), believes that interest rates will rise in the future, and wishes to swap from fixed into floating for the tenor of the instrument. The net return to the investor, taking into account all the cash flows in the bond and the swap, is LIBOR + 1.85%, as shown below:

Figure 7(b): 5-year IRS



Net return to investor = 2.75% - 0.90% + LIBOR = **LIBOR + 1.85%**

In principle, a swap contract could be tailored to exchange just about anything. In practice, most swaps fall into fall into four other categories aside from interest rate swaps: currency swaps, commodity swaps, equity swaps and credit swaps.

Options

Let us begin our introduction to the option product by distilling the product into the fundamental terms and conditions of the contract: an option contract conveys from the seller (or writer) to the buyer (or holder) of the contract the right, but not the obligation, to buy or sell a specific quantity of an asset at a specified price on or before a specified date. The writer will be short of an option position while the holder will be long of an option position.

Every time an options contract is agreed upon, the following terms need to be agreed upon by both parties in the transaction:

- a definition of the underlying asset
- whether the option is a put or a call
- the exercise price
- the maturity or time to expiration of the option
- whether the option is American-style or European-style

Let us clarify some of these definitions:

Underlying Asset	The asset which the option holder has the right to buy or sell. It includes interest rates, debt instruments, foreign currencies, stocks, stock indices and commodities as well as many others
Call	The right to buy a specified quantity of an underlying asset
Put	The right to sell a specified quantity of an underlying asset
Exercise Price	The price at which an option holder can exercise their right to buy or sell the underlying asset. Also known as the *Strike Price*
European Option	The option buyer has the right to exercise only at the time of the option's maturity
American Option	The option buyer has the right to exercise at any time until the option has expired

Having defined what s/he requires, the buyer will be quoted a price for this option by the writer. The price of the option will depend on a number of factors which we will study further later, but at this stage we observe that the cost will depend largely on whether the option is in-, at- or out-of-the-money. Let us clarify some more definitions:

Premium	The amount or fee paid by the option buyer to the option seller (writer)
In-the-money (ITM)	A term used to describe the relationship between the price or rate on the underlying, and the option's strike price. An ITM option is one whose strike price is more advantageous than the current market price of the underlying (eg. a call on a stock is ITM if the stock price is above the strike price of the option)
Out-of-the-money (OTM)	A term used to describe an option whose underlying is above the strike price in the case of a call, or below it in the case of a put. The more the option is OTM, the cheaper it is since the chances

of exercise are smaller.

At-the-money (ATM) This definition often causes confusion for inexperienced participants in the options market. Clarification is required as to whether the definition is relative to the spot price or the forward price. So we have at-the-money spot as the term for an option whose strike is set the same as the prevailing market price of the underlying, or at-the-money forward (ATMF) for an option whose strike is set at the same level as the prevailing market price of the underlying forward contract.

Intrinsic Value The amount by which an option is ITM. The intrinsic value of an ITM call on a stock is the amount by which the stock price exceeds the strike price. The intrinsic value of an ITM put, on the other hand, is the amount by which the strike exceeds the stock price. If the calls or puts are OTM, their intrinsic values are zero.

The Basic Option Transactions

Having defined the terms of an option contract, it is clear that there are four basic transactions that a party can agree to. They are:

- buy a call (acquiring the right to buy an asset)
- sell a call (selling the right to buy an asset)
- buy a put (acquiring the right to sell an asset)
- sell a put (selling the right to sell an asset)

The simplest way to understand how options work is with graphic illustrations. For the purposes of our explanation let us consider the asset to be a bond whose price will vary with changes in market interest rates, specifically a change in the interest rate for the tenor of our bond. We will use pay-off diagrams to show the four option transactions. Our co-ordinates are as follows:

- the horizontal axis represents the value of the bond
- the vertical axis shows the net return (as profit or loss)

We will use the following notation in our analysis:

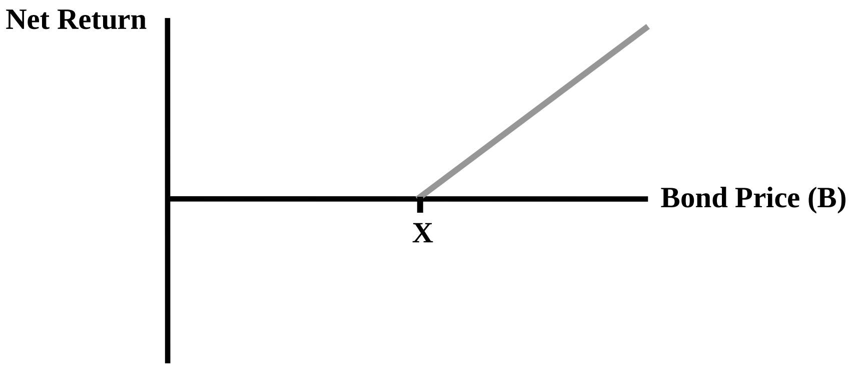
B	=	Bond price
C	=	Call value
P	=	Put value
X	=	Exercise price

Firstly, let us consider the value at expiration of a European-style call option on the bond.

- If, at expiration, the bond price is less than the exercise price (B<X), the option to purchase the bond is worthless (C=0) because it is possible to buy the bond in the market for less than X.
- On the other hand if the bond price is greater than the exercise price at expiration (B>X), the value of the right to buy the option is equal to the difference between the bond price and the exercise price (C=B-X). So if B>X at maturity, the call owner would buy the bond for price X and sell it in the market for price B, generating a profit of B-X.

This position is summarized in a pay-off profile graphed in **Figure 8**.

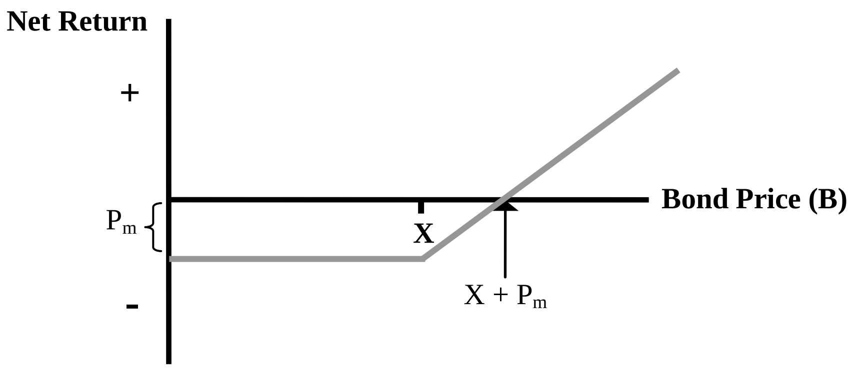
Figure 8: Pay-off profile



At the point of exercise (X), the call buyer becomes the owner of the bond and the line turns upward at a 45^o. The owner of the call option benefits at the expense of the seller of the call option. The pay-off to the option owner is B-X, and the pay-off (in this case a cost) to the option writer is the reverse, X-B. If however the bond price at expiration is less than the exercise price, the call option is worthless, so the pay-off to both parties is zero according to the graph.

However, we have so far ignored one very important item! From the option buyer's point of view so far s/he appears to be a no-lose situation! But we need to incorporate the fact that buying (or going "long") an option will incur the payment of a premium. At origination the buyer of the option pays the option writer a premium which will shift the pay-off profile for the buyer down as demonstrated in **Figure 9**. The premium is notated by P_m.

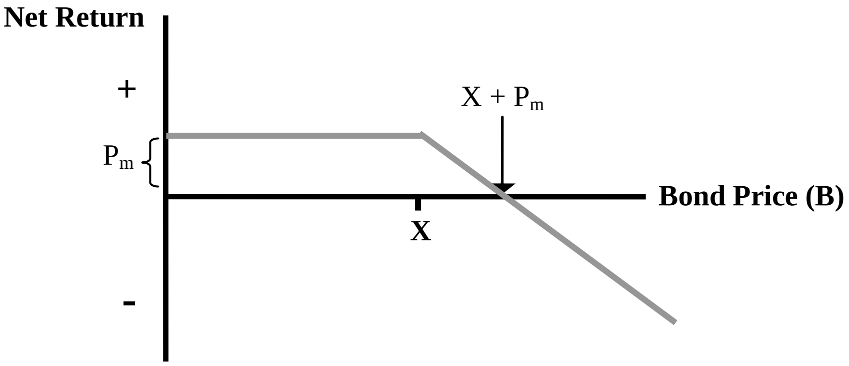
Figure 9: A long call position at maturity



Notice in this case that the line crosses the horizontal line at X + P_m (exercise plus call premium) which is the call buyer's break-even point.

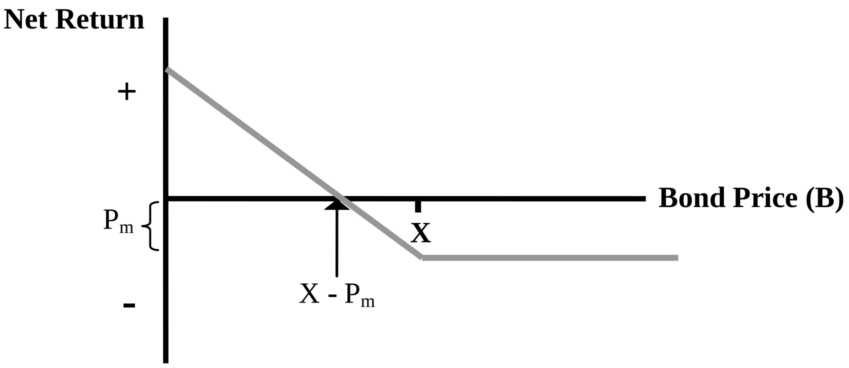
So how would the graph look from the call option writer's point of view? If we hold a mirror horizontally on the graph in Figure 11, we can see selling (or going "short") a call is the mirror image of buying a call. The writer collects the premium, P_m. However if the option is exercised, that line turns downward at a 45^o angle and crosses the horizontal line at X + P_m (exercise plus call premium) beyond which the writer incurs a loss as graphed in **Figure 10**.

Figure 10: A short call position at maturity



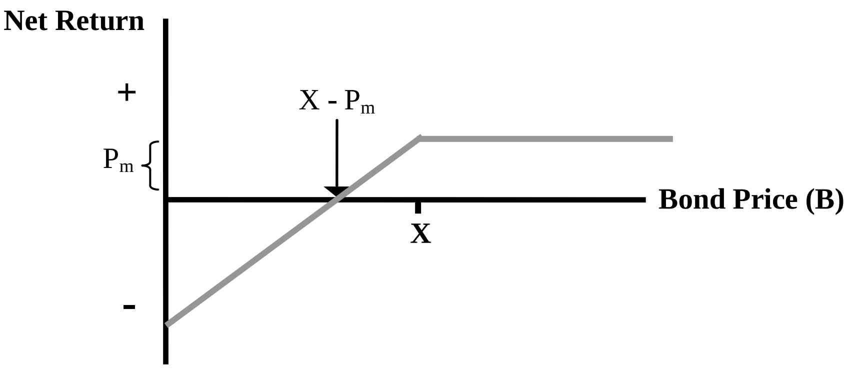
The graph for buying a put can be seen by placing a mirror vertically on Figure 11 as depicted in **Figure 11**. Notice that the line hits the horizontal axis at X **-** P_m (exercise minus the put premium) and continues downward until it breaks into a horizontal line at the exercise price (X). By acquiring the right to sell the bond, the put holder profits if the bond's value goes below the break-even point.

Figure 11: A long put position at maturity



If you place a mirror horizontally on Figure 13, you will see what selling a put looks like in **Figure 12**.

Figure 12: A short put position at maturity



Let us look in more detail at the motivations of clients who wish to enter into such transactions.

Option Applications

Let us consider the following applications:

- Insurance or hedging
- Yield enhancement
- Trading

Insurance or hedging
An analogy can be made with the insurance policy on your car. You pay insurance premiums on your car even if you consider yourself a good driver because you can't do anything about car thieves or other drivers. Similarly option premiums can be viewed as the cost of purchasing protection for one's assets.

Example: An investor, who owns a bond with a 3-year maturity, would like to buy a put on the bond price because s/he wants protection in the event that interest rates rise and the value of their bond will fall. Buying a put option on the bond insures a minimum sale price, and consequently, a minimum return on the investment.

Yield enhancement
Options can be used to provide an income or return when the market is expected to move in such a way that an option sold will not be exercised.

Example: Another investor who owns a bond but believes that interest rates will be steady to slightly higher could sell a call on that bond that s/he already owns in order to generate premium income over and above the coupon interest already being paid on the bond.

This is an example of a **covered call**: the call writer sells calls on assets s/he already owns. **Uncovered** or **naked calls** are written on assets that writers do not own but intend to acquire if the buyer exercises the option. This latter strategy is obviously a high-risk proposition for the writer.

Trading
Options may be used to augment traditional trading tactics. We have mentioned options allow an investor to take a leveraged view.

Example: An investor is convinced that interest rates will fall and that bond prices go up over the next 3 months, but does not have the cash to purchase the bond. Paying a relatively small option premium allows the investor to take a leveraged trading position.

In addition, in some markets it may be difficult if not impossible for non-institutional participants to go short a particular asset in the cash market, but this may be "synthetically" achievable through options.

Furthermore options also permit traders to take a view, not only on a directional move, but also on the price volatility of an asset.

Option Valuation

Although options in various forms have been around for centuries, a formal pricing model did not appear until the early 1970's. Fisher Black and Myron Scholes made a major breakthrough by deriving a differential equation for pricing European put and call options on a non-dividend paying stock. Since then the original model has been modified to cope with a wide range of assets and nowadays many institutional players use their own proprietary models. Nevertheless the fundamentals of valuation have not changed and it is to the Black-Scholes model we now turn. While a detailed analysis of the Black-Scholes formula is beyond the scope of this workbook, we will briefly describe the information the formula gives us.

Professors Black and Scholes determined that the value of an option is a function of the following variables:

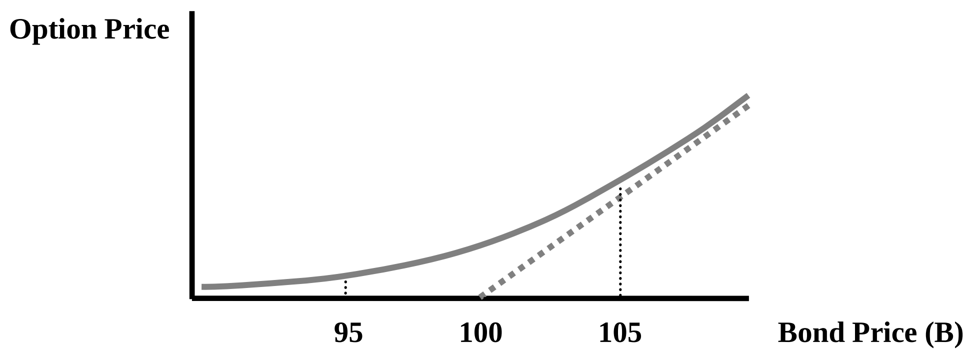
- exercise price of the option
- asset price/rate
- time to maturity
- prevailing interest rate
- volatility of the asset

Let us consider the analysis in terms of an option on a security: a 12-month European call on bond struck at, say, 100. Let us also assume that the strike of 100 is also equivalent to the 12-month forward price of the security. We could say that the call is an at-the-money forward (ATMF) option.

Generally, the most important influence on the option's price will be price of the bond itself because if the price is way above or way below the strike of 100, then the other factors will have little influence. Its dominance is obvious on the expiry date of the call for on that day only the bond's price and the strike determine the call's value, and the other factors have no bearing at all. At this time the call is only worth its intrinsic value. When the option is either OTM or ATM, the intrinsic value is zero.

Prior to expiration, the call's total price will consist of both intrinsic value plus what is called its *time value.* (It should be clear that before maturity, even when the call is OTM, the option writer will always charge a premium because there is a possibility between now and maturity, the price of the bond will rise above 100. The resultant price curve for the call takes the shape graphed in **Figure 13**.

Figure 13: The call option price curve



We observe from the graph that the time value premium is worth most when the option is right ATM, whereas when the bond's price is way below the strike, time value is small, or when it is way above the strike, then the option's value is virtually all intrinsic value.

Note that this curve demonstrates the value of option at different bond prices with 6 months to expiry, but as that time gets shorter and maturity approaches, that line will move lower and closer to the "hockey stick" profile until on expiry day when it completely merges with the intrinsic value line. That is, the option is only worth its intrinsic value at maturity.

So, intrinsic value is easy to derive (being the difference between the spot rate and the exercise price) but how do we assign a value to this thing we call "time value"?

We can think of an option's time value as being a function of probabilistic expected returns. We can, for instance, make an assumption right now about the path of the bond price over the next 6 months. We will question and address our assumption shortly but if for the time being we believe that - with the bond price now at 100 - there is an array of possible bond prices in 12 months' time with the most likely rate being 100 (which happens to be the 12-month forward price of the bond). The further away we move from that rate, the less likely the bond price is likely to get there. We can plot for a range of bond prices and the probability of each of those rates shown in **Figure 14**.

Figure 14: Probabilistic distribution for the bond price in 12 months

Bond price	**Bond price in 12-mo**	**Probability**
	120.00	5%
	110.00	25%
100.00	100.00	40%
	90.00	25%
	80.00	5%
		100%

In **Figure 15** we take the idea a little further as follows:

- We can calculate the call pay-off at each of those levels of the bond price for a 100 strike (ie. the intrinsic value at maturity) (*Column 2*)
- We can calculate the *Expected pay-off* for each level of the bond price by multiplying *Column 1* by *Column 2*, to give us the numbers in *Column 3*. If we then sum those numbers in Column 3, we get the total expected (future) pay-off under the call (ie. 3.50).
- This last number represents the future value of the option, so in order to calculate the up-front cost, this number needs to be discounted back over 12 months to calculate a fair market price today. This final number is, of course, the premium for the option.

Figure 15: Call price calculation

		Column 1	*Column 2*	*Column 3*
Bond Price	**Bond price in 12-mo**	**Probability**	**Pay-off (X = 100)**	**Expected pay-off**
	120.00	5%	20.00	1.00
	110.00	25%	10.00	2.50
100.00	100.00	40%	0.00	0.00
	90.00	25%	0.00	0.00
	80.00	5%	0.00	0.00
		100%		**3.50**

The sum of the expected pay-offs would have to be present valued in order to determine the fair market value today, ie. the option premium

But, the question remains: on what basis do we make our assumptions about:

- the rate of 100.00 being the most "likely" price in 12 months, and
- the *range* of prices around 100 in 12-months time?

For the reasoning we defer again to the work of Black and Scholes.

- They proved after much analysis that the probability distribution of the underlying for some future date centered around the forward rate for that date. In other words the most likely price was the no-arbitrage rate as suggested by the standard forward pricing model. As with virtually all financial market instruments, the forward price is a function of interest rate differentials, the two rates whose differential is considered being the cost of borrowing money versus the bond coupon over the time period concerned. We will assume that both rates are identical, so there is no net cost-of-carry, and therefore the forward bond price is the same as the spot price ie. 100.
- Secondly they defined the uncertainty range surrounding that forward price in terms of volatility.

Our model is a simplistic representation of the Black-Scholes model for their calculations were based on tracking probabilistic payoffs for a *continuous* set of possible forward rates rather than a discrete number as in our case.
An analysis of the pricing model which relies on probability distributions can best be demonstrated by means of a traditional bell-shaped distribution curve whose two mathematical parameters are the :

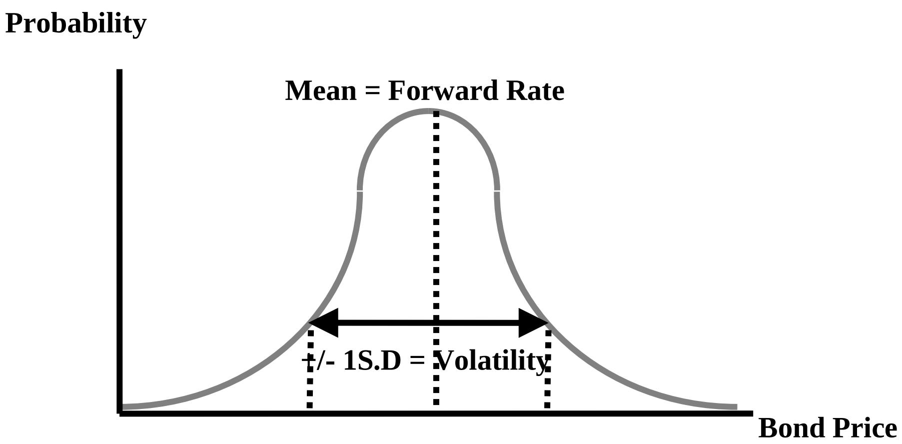
- mean (or average)
- standard deviation (or range of possible prices)

For the purposes of option pricing these statistical terms can be restated in financial terms as, respectively, the

- forward rate
- volatility (determined as a +/- one standard deviation around the forward rate)

Figure 16 summarises these concepts.

Figure 16: Bell Curve



Volatility

What is volatility?
Intuitively it can be defined as the likely trading range of the asset. It is defined:

- as an annual percentage
- on a +/- *one* standard deviation basis around the forward rate (ie. with a confidence level of approximately 68%)

What does it mean?
Let us take an example:

Example: Assume a security is being quoted at 100 for delivery one year forward and that the volatility of this bond is being quoted at 10%. That is to say, the market believes this bond is likely to be trading within a range 10% up from its forward price and 10% down from its forward price one year from today with a 68% level of confidence (ie. that the likely trading range is 90 - 110)

Conclusion

Thus concludes our description of a number of the basic derivative products. Having started with a description of the simplest of derivatives, being a single-period forward product in the form of an FRA, we then looked at multi-period forward contracts in the form of swaps, and concluded with a description of options. For each product, we have had the opportunity to review the conventions, look at some applications and present some intuitive pricing techniques.

Within the broad family of derivatives, there are many much more complex structures, but the fact remains that whatever the product, it can be decompounded into one or more of the basic products that we have examined in this text.